

STIMULATED RADIATION OF PARTICLES IN CRYSTALLIC UNDULATORS

R.O.Avakian, L.A.Gevorgian, K.A.Ispirian^{*)} and R.K.Ispirian

Yerevan Physics Institute, Brothers Alikhanian 2, Yerevan, 375036, Armenia

Abstract

It is investigated the stimulated radiation which arises when the density and energy of the beam of relativistic particles channeled in microscopic crystalline undulators (CU) obtained by the application of transverse ultrasonic (US) oscillations to single crystals, exceed certain values. For positron beams expected at future linear colliders and real CU it is given the results of numerical calculations for spontaneous and stimulated radiation.

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The problems connected with the construction of x-ray and gamma lasers remain still unresolved (see [1]). The possibility of the production of the stimulated channeling radiation has appeared to be almost hopeless since it requires particle beams of very high density for which the crystals will be damaged (see [2]). Nevertheless, arrangements are under construction at present for production of high brightness x-ray beams in which the self amplification of the spontaneous emission (SASE, the operation principles of which are proven for centimeter and micron wavelengths) will take place (see [3]).

Looking for intense radiation sources in the works [4,5] it has been considered the radiation produced by positively charged particles in CU. The influence of the medium polarization on the spontaneous radiation produced in CU has been taken into account in [6]. The estimates [5] show that at high positron beam densities this radiation becomes stimulated. Taking into account the above practical interest this work is devoted to the investigation of various characteristics of the CU stimulated radiation taking into account

^{*)} e-mail:ispirian@vx1.yerphi.am

the beam and medium density effects.

Using the formula (4) of the work [6] after necessary folding one can show that for a positron beam with gaussian distribution of the energy E or of the Lorentz factor $\gamma = E/mc^2$ of the particles around $\bar{\gamma} = \bar{E}/mc^2$ with dispersion σ , the spectral distribution of the number of photons of the spontaneous radiation produced by a channeled particle in a CU with length L has the form

$$\langle \frac{dN}{d\xi} \rangle = Df(\xi)F[X(\xi)], \quad (1)$$

where

$$\begin{aligned} f(\xi) &= \frac{1}{2} \left[\left[(1 + \eta^2 - \eta_c^2)\xi + \frac{1}{\xi} - 1 \right]^2 + 1 \right], \\ F[X(\xi)] &= 1 - \text{erf}[X(\xi)], \\ X(\xi) &= \frac{1}{\sqrt{2}\sigma} \left[\frac{\xi}{p\eta\sqrt{(\xi - \xi_1)(\xi_2 - \xi)}} - 1 \right], \\ \xi_{1,2} &= \frac{1}{1 \pm \sqrt{1 - \eta_c^2}}, \\ \eta_c &= \sqrt{1 - (\gamma_0^2/\bar{\gamma})^2}, \end{aligned}$$

$D = \pi\alpha\eta^2 L/l$, $\alpha = 1/137$. As in [6] $\eta = \sqrt{2}\pi A/\lambda_p$, $\lambda_p = 2\pi c/\omega_p$ plasma wavelength of the CU, $\gamma_0 = \omega_p/\Omega_p = l/\lambda_p$, $\Omega_p = 2\pi c/l$, $\lambda_0 = l/\gamma_0^2 = \lambda_p^2/l$ is the wavelength around which the narrowing of the spectrum takes place due to the polarization of the medium [6], while A and $l = v_{us}/f_{us} = 2\pi c/\Omega$ are the amplitude and the period of the CU obtained as a result of application of transversal US oscillations with f_{us} and velocity v_{us} . Instead of x and k of the work [6] it is introduced new dimensionless energy of photon $\xi = \omega/\Omega\gamma_0^2 = \lambda_0/\lambda$ and positron $p = \bar{\gamma}/\gamma_0 = 1/\sqrt{1 - \eta_c^2}$. The radiation takes place when $\gamma \geq \gamma_{thr} = \gamma_0/\sqrt{1 - \eta^2}$ or $p \geq p_{thr} = 1/\sqrt{1 - \eta^2}$.

The spontaneous radiation spectral distributions of positron beams, calculated with the help of the formula (1), differ from the corresponding curves of ref. [6] by their smooth boundaries due to the energy spread of the beam particles.

In the validity limits of the one dimensional (1D) FEL theory in which the diffraction is not taken into account, following the works [3,7] and neglecting the angular divergence

of the beam with density n , it can shown that for low gains of the stimulated radiation when the gain $G \leq 1$ one has

$$G(\xi) = G_0 \frac{p}{\sigma} \varphi(\eta) \psi[X(\xi)], \quad (2)$$

where

$$\begin{aligned} G_0 &= \frac{(2\pi)^3 r_0 n L^2}{\gamma_0^3}, \\ \varphi(\eta) &= \frac{\eta^2 (1 - \eta_c^2 + \eta^2)^2}{1 + \sqrt{\eta_c^2 - \eta^2}}, \\ \psi[X(\xi)] &= \exp[-X^2(\xi)] F[X(\xi)], \end{aligned}$$

$r_0 = e^2/mc^2$ is the electron classical radius.

For $\eta \rightarrow 0$ the function $\varphi(\eta)$, and therefore G goes to 0, while for $p \gg 1$ and $\eta \rightarrow \eta_c$ the function $\varphi(\eta) \rightarrow 1$, and G achieves its maximal value. The function $\psi(\xi)$, therefore G too have their maximum when

$$\xi_0 = \frac{1}{1 - \sqrt{\eta_c^2 - \eta^2}} \quad (3)$$

with an half width of the stimulated radiation spectral distribution

$$\Delta\xi \approx \sqrt{1 - \eta^2} \xi_0^2 \sigma. \quad (4)$$

The maximal value of G_{max} is obtained at $\eta \rightarrow \eta_c$ ($\xi_0 = 1$, $\Delta\xi = \sigma/p$) and $p \gg 1$:

$$G_{max} \approx \frac{p}{\sigma} G_0 = n/n_c, \quad (5)$$

$$n_c = \frac{\sigma \gamma_0^3}{(2\pi)^3 r_0 p L^2} = \frac{\gamma_0^4}{(2\pi)^3 r_0 L^2 \bar{\gamma}}, \quad (6)$$

where n_c is the beam density for which the gain is of the order of 1. The further increase of the density results in the growth of the stimulated radiation intensity.

When σ decreases, $\Delta\xi$ also decreases and the values of G and G_{max} increase. Let us note that in contrast to the usual FELs without filling medium γ_0 enters into the expressions for G_0 and n_c instead of γ that results in essential increase of the gain as in the case of gas loaded FELs [8].

In the nearest future it will be obtained beams with $\sigma \leq 0.001$ and particle density $n = 1.0 \cdot 10^{21} \text{ cm}^{-3}$ at $\overline{E} = 250$ and 50 GeV in the interaction point of the e^+e^- collider TESLA [9]; $n = 1.8 \cdot 10^{18} \text{ cm}^{-3}$ at $\overline{E} = 10 - 25$ GeV in the TESLA x-ray FELL undulator with $L = 87 \text{ m}$ and $n = 6.4 \cdot 10^{16}$ at $\overline{E} = 1$ GeV in the TTF TESLA undulator with $L = 27 \text{ m}$ [10]. Let us note that at $\overline{E} = 1$ and 10-25 GeV the beams will be electron beams. However, corresponding positron beams can be produced after the TESLA launching, and the beam densities can be increased by 1-2 orders with the help of focusing because the length of CU is much shorter than the length of SASE FELs.

The necessary parameters and results on the stimulated radiation calculated for these beams with the help of the formulae (2)-(6) for quartz CU (see [6]) are given in Table. After the values of the energy (first column) the second column gives the CU thicknesses which are permitted by dechanneling and other technical reasons. The third column shows the desired maximal numbers of the CU periods. These values of E , L and N determine f_{us} (fourth column), l , Ω , γ_0 , p , η_c and n_c (fifth column). The sixth column gives the chosen values of the US amplitudes which provide the necessary values of η , ξ_0 , i.e. the stimulated radiation photon energies $\omega_0 = \xi_0 \Omega \gamma_0^2$ (seventh column). In the next two columns it is given the relative half widths $\Delta\omega/\omega = \Delta\xi/\xi_0$ and the gains calculated with the help of the formula (2). As it follows from the Table besides the case $\overline{E} = 1$ GeV, even for densities $n \approx n_c \approx 10^{13-3}$, much less than the ones of the future beams one, can obtain values of G of the order of 1. It is clear that values $G \gg 1$ can be obtained with the help of beams with higher density, and intense monochromatic beams can be produced with photon energies $\omega_0 = 33; 55$ and 100 keV, i.e. with energies much higher than the photon energies which will be produced at the linear accelerators SLAC and TESLA.

The intensity of the radiation from CU per beam particle is equal to the product of $(G+1)$ and spontaneous radiation spectrum (1). It is clear that for $G \leq 0.2$ there is little difference between the spectra of the spontaneous and stimulated radiations at the CU exit. Therefore, for small values of G the stimulated radiation in CU will be of interest if x-ray cavities (see, for instance [1]) will be created in which the losses per one cycle is less than G , and there is radiation photon storing from the consequent bunches of positron

pulses. The photons produced and stored in the cavities by the bunches of positron beam macropulses can serve as "X-ray target" for the study of many processes [11].

The gain and the spectra of the stimulated radiation for high gain FELs with $G \gg 1$ are considered within the 1D FEL theory in many works (see [3, 12, 13]). In conclusion without considering these and other problems connected with the multiple scattering (the CU thickness is less than the dechanneling length, so that the particles remain channeled), with the US amplitude and frequency spread, with diffraction and with particle density modulation and beaming when $G \gg 1$, let us estimate the possibility of the SASE process in CU. The Pierce parameter and the length for e time enhancement of the intensity of SASE FELs can be written in the form $\rho = 2.82 \cdot 10^{-5} (K^2 l^2 n)^{1/3}$ and $L_g = 3.26 \cdot 10^3 (K^{-2} l n^{-1})^{1/3}$, respectively, where $K = 2\pi\gamma A/l$. The values of ρ and L_g for the above considered CU and beams with the expected densities are given in the last columns of the Table, respectively. As it follows from such a direct application of the SASE theory stimulated radiation with intensity exponential growth and saturation processes can also take place in CU. In future the use of nanotubes with much smaller l and larger dechanneling lengths [14] can make easier the problem of the production of SASE sources of intense x-ray beams.

References

- [1] V.I.Visotski,R.N.Kuzmin,Gamma Lazeri,Publ. House of Moscos State University, Moscow, 1989.
- [2] V.A.Bazilev, N.K.Zhevago, Izluchenie Bistrikh Chastits v Veshchestve i vo Vneshnikh Polyakh, Nauka, Moscow,1987; V.N.Baier,V.N.Katkov, V.M.Strakhovenko, Elektromagnitnie Protsesi Pri Visokoy Energii v Orientirovanikh Kristallakh, Nauka, Novosibirsk, 1989.
- [3] W.B.Colson, Nucl. Instr. and Meth. A272,386(1988); A407,26(1998); Conceptual Design of a 500 GeV e^+e^- Linear Collider with Integrated X-Ray Laser Facility, Eds. R.Brinkman et al, DESY 1997-048/ECFA 1997-182, 1997.
- [4] V.V.Kaplin, S.V.Plotnikov, S.A.Vorobyov, Zh. Tekh. Fiz.,50,5(1979).
- [5] A.V.Korol, A.V.Solovyov and W.Greiner, J. Phys., G: Nucl. Part. Phys. 24,L45(1998).
- [6] R.O.Avakyan, L.A.Gevorgyan, K.A.Ispirian, R.K.Ispirian, Pisma Zh. Eksp. Teor. Fiz., 68, 437(1998).
- [7] L.A.Gevorgian and P.M.Pogosian, Nucl. Instr. and Meth., A351, 565(1990).
- [8] L.Gevorgian, Resonance Hard Radiation in a Gas Loaded FEL, Abstracts of the 17th Intern. Free Electron Laser Conf., August 21-25, New York, 1995.
- [9] J.Rossbach, DESY M98-11,1998,To be published in Proc. LINAC 98.
- [10] D.Trines, DESY M98-11,1998,To be published in Proc. LINAC 98.
- [11] K.A.Ispirian, M.K.Ispirian, R.I.Ispirian, Pisma Zh. Eks. Teor. Fiz., 58, 175(1993).
- [12] E.L.Saldin, E.A.Schneidmiller, M.V.Yurkov, Phys. Rep., 465,189(1995).
- [13] C.Pellegrini, Nucl. Instr. and Meth., A272, 364(1988).
- [14] L.A.Gevorgian, K.A.Ispirian, R.K.Ispirian, Pisma Eksp. Teor. Fiz., 66, 304(1997).

Table

E	L	N	f	$n_c 10^{-13}$	A	ω_0	$\Delta\omega/\omega$	G	$\rho 10^3$	L_g
GeV	cm	-	MHz	cm^{-3}	nm	keV	%	-	-	cm
250	10	1000	57	1.5	8.35	101	0.01	0.68	1	0.85
50	5	1000	114	1.9	8.26	54.7	0.02	0.51	2	0.25
25	2.5	1000	228	0.94	7.94	33.1	0.05	0.7	0.3	0.84
1	0.2	200	570	93.4	7.8	13.6	0.06	0.007	0.3	0.35